Fluid-structure interaction model for a vibrating cantilever

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Abstract

Fluid - structure interaction in a small scale cantilever system with one end clamped is modelled with the commercial Linflow program that solves the interaction problem with the Boundary Element Method. The air loaded resonance frequencies of the vibrating cantilever are calculated numerically and compared to theoretical and experimental results.The flow and pressure fields around the structure are evaluated and the feasibility of the results checked. The current version of Linflow appears to be a feasible program in solving relatively small scale coupled linearized problems of fluid-structure interaction.

Key words: Vibrating cantilever, FEM, BEM, Fluid-structure interaction.

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1 Introduction

In the studies of micromechanical systems three complementary approaches, namely experimental, theoretical and computational, can and should be used [1, 2]. Of these the experimental approach, though realistic, can be very expensive and sometimes impossible to arrange and do. For example in small systems the measurement devices can substantially disturb the system behaviour and the measured subject looses its characteristics. In the theoretical approach, however, the governing physical equations must often be greatly simplified and even transformed into linear form in order to derive analytical solutions, and thus a lot of information may be missed. Many of these difficulties can be bypassed in the computational approach, which can be used to describe even complex nonlinear transient systems. However, it must be remembered that computational solutions still are only good approximations of the real problems, that can contain errors caused by cut-off of the continuous functions. Also the convergence of the numerical solution may be a problem and the solution may require a very long time to reach. Thus the governing equations may also in this case need some simplification, e.g. linearisation, for faster convergence and solution speed-up.

In general, the fluid-structure interaction is a complex problem. It can be solved for example by simultaneous or partitioned integration [3]. In the former method all the governing equations are solved at once, which is mathematically a heavy procedure. The latter method uses a common physical quantity, e.g. pressure, to connect separately solved fluid and structure parts together. These methods may demand several hundreds of iterations before converging to solution.

In this article the feasibility of Linflow program for solving micro-scale problems is studied. Linflow is a solver for fluid-structure interaction problems, based on Boundary Element Method, BEM (See e.g. [4]). The structural analysis of the problem is solved using Ansys, which is based on Finite Element Method, FEM. Linflow evaluates the transient flow and pressure fields on the faces of the structure assuming that no boundary layer behaviour for the fluid exists, that is, the fluid is inviscid, and that the flow is potential, i.e. , irrotational. In order to study the feasibility of Linflow program in describing fluid - structure interaction we have assumed a cantilever system, with one end free to vibrate while the other end is clamped. In this approach the model structure is defined by Ansys procedure and we assume the material of this structure to be silicon. To the extension of the free end of the system we have assumed a rigid plate of the same thickness as the cantilever. This plate is fully supported by the side walls and thus it is wider than the vibrating cantilever. The cantilever and the rigid plate are above a thin air-film defined by the floor and the surrounding walls. A schematic picture of the system without side-walls is presented in Figure 1. The longitudinal scales of the structure vary from the thousands of micrometers of the cantilever length down to few micrometers of the air gap between the cantilever and rigid plate. The air-film thickness is a few hundred micrometers.

The analysis procedure solves eigenmodes of a vibrating cantilever with the FEM in the Ansys package, and damping and flow fields in the system around the cantilever with the BEM in Linflow. The effect of the fluid on the vibrating cantilever is evaluated by studying the frequency changes in the vibration, induced by the fluid pressure. The variations of the flow shapes and pressure field around the cantilever as functions of the air gap between and air film thickness below the cantilever and the rigid plate are studied.

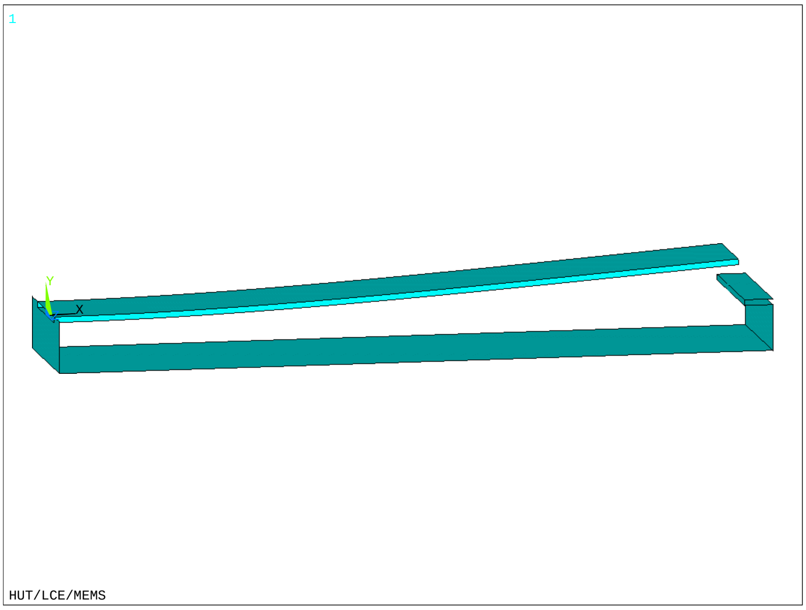

Figure 1: Schematic illustration of the cantilever and the system around it. One end of the system is clamped. The side walls are left out of the picture. Displacement of the first mode.

2 Theory

There are a number of devices that utilize longitudinal and transversal vibrations in cantilevers. According to the basic theory it is assumed that the transversal displacement of the vibrating cantilever is relatively small compared to its length and thus limited to linear regime. In an ideal case there are no external forces acting on the cantilever. The motion of the unloaded cantilever can be expressed by the following wave equation in the case of two dimensional thin cantilever [5]:

$$\frac{\partial^2 y}{\partial t^2} = \kappa^2 c^2 \frac{\partial^4 y}{\partial x^4}. \qquad (1)$$

Here y is the displacement at position x, c is the velocity of transverse sound wave in the medium ($5.84 \cdot 10^3$m/s for silicon), κ is the geometrical cross-section parameter and t is the time. In the case of a rectangular cantilever, κ can be expressed as a function of the thickness τ of the cantilever, $\kappa = \tau/\sqrt{12}$. At the clamped end of the cantilever all the degrees of freedom are restricted to zero. The solution for this small amplitude problem is:

$$y(x,t) = cos(\omega t + \phi)\left[B\left(cosh(\frac{\omega x}{\nu}) - cos(\frac{\omega x}{\nu})\right) + C\left(sinh(\frac{\omega x}{\nu}) - sin(\frac{\omega x}{\nu})\right)\right], \qquad (2)$$

where ω is the angular frequency, t the time, ϕ the phase angle and $\nu = \sqrt{\omega c \kappa}$ is the phase speed. B and C are constants that define the amplitude of the vibration. This equation is the product of time-dependent and coordinate-dependent factors. The first factor of the equation defines the time-dependent shape of the cantilever, and the second factor the coordinate and mode dependent shape. Then the maximum velocity at the free end is as follows

$$\dot{y}(L, \tilde{t}) = \pm\omega A = \pm 2\pi f_{\mathrm{m}} A, \tag{3}$$

where A is the amplitude of the vibration and f_{m} is the eigenfrequency of the vibrating cantilever. A cantilever with length L reaches the maximum velocity when $\tilde{t} = (\pi/2 + n\pi - \phi)/\omega$, $n = \{0, 1, 2, \dots\}$. Magnitude of the amplitude is derived from the selection of B and C in equation 2. Hence the eigenfrequencies for the cantilever clamped at one end are:

$$f_{\mathrm{m}} = \frac{\pi c \kappa}{8L^2}\left(1.194^2, 2.988^2, 5^2, \dots\right), \tag{4}$$

where m is the mode number having values of m$= \{1, 2, 3, \dots\}$. The used boundary conditions limit the allowed modes of free vibrations of a finite cantilever to a discrete set of frequencies. It is worth noticing that overtone frequencies are *not* harmonics of its fundamental.

The cantilever can be loaded with a fixed mass and/or with continuous fluid. In our case the free space around the cantilevers is filled with fluid, specifically with air. The deflection of the cantilever generates flow and pressure fields in the air. Their behaviour can be expressed with a continuum model described by the continuity and Navier-Stokes equations for incompressible (density ρ is constant) flow:

$$\nabla \cdot \vec{v} = 0 \tag{5}$$

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{\nabla p}{\rho} + \frac{\mu}{\rho}(\Delta \vec{v} + \frac{1}{3}\nabla(\nabla \cdot \vec{v})). \tag{6}$$

Here $\vec{v}$ is the velocity vector of the flow, p is the pressure and μ is the viscosity of the flow. The nature of the flow field can be evaluated with its Reynolds number, Re, such that small Re indicates laminar flow field whereas large Re indicates turbulence type of flow field. The numerical value for the transition is defined by the fluid density, charasteric velocity of the fluid flow, v_∞, geometrical length of interest, L, and fluid viscosity, $Re = \rho v_\infty L/\mu$. For air $\rho/\mu \sim 10^4 \mathrm{s/m^2}$. In the structural scale where the amplitude of the vibration reaches at most some hundred micrometers, the Reynolds number depends linearly on the velocity. For small vibration regime the velocities of the first eigenmode even in their maxima remain relatively small, and thus the flow can be assumed laminar.

The validity of the continuum model can be estimated with the Knudsen number, Kn. It determines whether the scales of the gas filled structure are such, that a continuum model is valid or whether the molecular behavior of the gas should be taken into account in the governing equations of the dynamics. Knudsen number is defined by comparing the molecular properties of the gas to the characteristic length of the structure, i.e. $Kn = \lambda/\ell$. Here λ is the mean free path for gas molecules and ℓ is the distance between walls. In general, for Knudsen number $Kn < 0.1$ the continuum equations are valid and for $Kn < 0.001$ the no-slip condition for flow is held true [6]. The mean free path of air molecules is $\lambda = 0.065\mu$m. Here Kn for the cantilever system is less than $Kn \sim 10^{-2}$ for the smallest air gap. Thus the flow is within continuum regime and varies within no-slip and slip flow regime. Linflow assumes that the flow is in the no-slip flow region, and does not take slip flow effects into account. However, in this model the the error due to this simplification will not be considerable due to the quite simple geometry of the model.

The flow is also assumed potential, i.e. , irrotational, and thus the velocity vector can be stated with velocity potential ϕ:

$$\nabla \times \vec{v} = 0 \Rightarrow \vec{v} = \nabla\phi. \tag{7}$$

In Linflow the solution is based on complex potential [7], and thus also the velocity and pressure are stated with complex variables. In Linflow 1.0 the boundary layer effects are not taken into account, and the fluid is simplified to be inviscid, $\mu = 0$. Since the slip effect is not assumed to be important in this model structure, the boundary conditions for the inviscid fluid are

$$\vec{n} \cdot \vec{v}|_{wall} = \vec{0} \tag{8}$$

$$\vec{v}|_{t=0} = \vec{0}. \tag{9}$$

The gas in this aeroelastic model is also assumed perfect and isentropic.

3 Modelling procedure

The main steps of the modelling process are shown in Fig. 2. It begins with definition of the model structure in Ansys definition procedure. As described above the model consists of a solid vibrating silicon cantilever surrounded by a rigid system of walls, floor and a fixed plate, which are modeled as areas. The length of the vibrating cantilever, which is clamped from one end to a wall, is $4546 - 4800\mu$m depending on the analysis to be made. A 198μm long rigid plate, which is clamped to another wall, is as an extension to it. The air gap between the cantilever and plate varies from 2μm to 256μm for different analyses such that the total length of the model between the end walls is constant 5000μm. The thickness of the cantilever and plate is 40μm and the air film height between the cantilevers and the floor under them is 340μm. The widths of the model vary from the 600μm of the cantilever width to 1000μm of the wall and rigid plate width. The floor and end walls are connected to the side walls of the system.

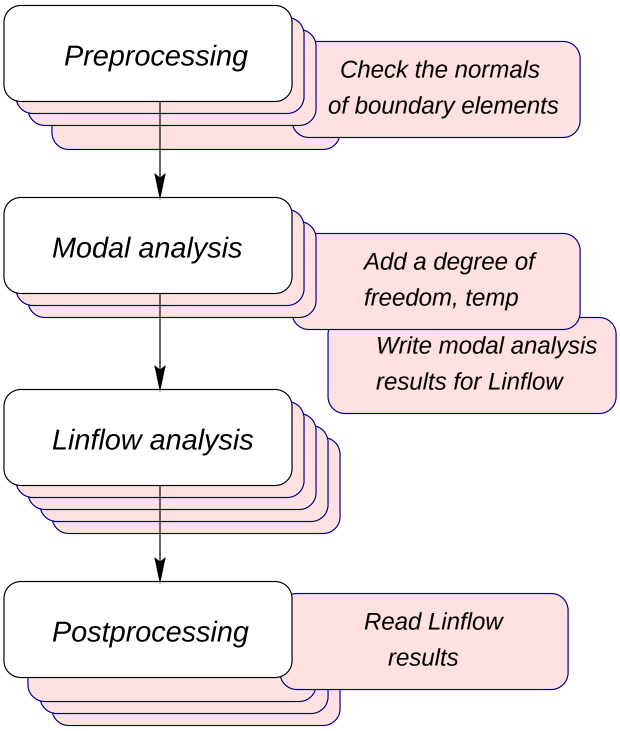

Figure 2: Block diagramme of the modeling procedure. The special steps required for Linflow analysis in the other procedural steps are shown beside.

The Ansys procedures are based on FEM while the Linflow procedure is based on BEM. The element types in the Linflow version used in this work are restricted to 4-noded boundary elements, which on the hand are not appropriate elements for the three dimensional structural analysis. (Linflow 1.1 include both 3- and 4-noded elements.) Thus the volumes and areas of the model are converted to different finite elements. The vibrating cantilever itself consists of solid elements, which represent the silicon structure. The areas around and covering it are created by very flat shell elements (boundary elements) with the same material parameters of silicon. The areas that are to be included in Linflow analysis must be meshed before other parts of the model to result consecutive element numbering beginning from 1. All these areas must also be of the same element type and the normals of these elements must point into the flow area of the system.

The solution procedure begins with Ansys modal analysis. After that the boundary conditions are set, the analysis is performed and the eigenmodes of the vibration solved giving same the results as equation (4). Then the results are written for Linflow in the postprocessor and a new degree of freedom, temp, is added for the boundary nodes in the definition procedure to represent the velocity potential defined in equation (7). The procedure is then moved to solution mode, where the fluid parameters and initial and boundary conditions for the flow are set. Linflow evaluates the variables only on the boundary elements. Transition procedure from the pure structural model to added air damped structural model is bypassed and the fluid-structure interaction is evaluated in a steady-state situation, where the air damping may be taken as a constant mass added on the cantilever. The analysis for the transient flow around the harmonically vibrating cantilever is performed by superimposing the dynamic flow field on the steady flow field. In order to predict the flow and pressure fields also in the fluid volume, the model can be cut by a meshed plane and the flow variables are evaluated in these external nodes in Linflow. Linflow solution procedure of models of some thousands of elements takes at most a few hours depending on the performance of the computer system [8].

4 Results and Discussion

First we discuss the modelling results of the eigenfrequencies of our cantilever system. The eigenfrequencies of the first and second modes for 40μm thick cantilever were found to vary from 1.7kHz to 2.0kHz and 10kHz to 12kHz, respectively, such that the shorter the cantilever is, the higher is the eigenfrequency. These results can be compared with resonance frequencies of a similar small scale cantilever system studied experimentally by Hietanen *et al.* [9]. We found that the eigenfrequencies solved by Ansys structural analysis package fit well the theoretical (Eq. 4) and experimental eigenfrequencies.

Next we looked at the effects of air damping in the system. In our Linflow model the ambient pressure of 1025 mbar was found to reduce the eigenfrequencies by $0.3 - 0.4\%$ compared to the values in vacuum. This is to be compared with the experimental results which yielded a decrease of $0.6 - 0.9\%$ in the eigenfrequencies for similar ambient pressure change. Thus the magnitude of damping is predicted relatively well. Higher ambient pressure increases linearly the rate of damping for all the modes. Also thinner air film below the cantilevers reduces the eigenfrequencies and thus the rate of damping becomes higher. However, the nonlinear effects which should occur with very thin air films are missed due to the linearity assumption of Linflow.

The effect of the size of the air gap between the vibrating cantilever and the rigid plate can be evaluated from the results of the Linflow analysis. With small air gaps (2μm-...) the damping effects are more intense than with larger air gaps (...-256μm), and the eigenfrequencies are reduced more. This is due to the difference in the size of the free cross-sectional area between the cantilever and the plate, i.e. , air gap length. The rigid plate blocks the increasing part of the flow as function of diminishing area and thus less air volume is able to move from one side of the cantilever to another. For the velocity field to respond to the cantilever displacement induced volume and pressure changes the air flow is accelerated through the small areas round the cantilever. The smaller the free flow area is, the larger the air flow velocities are.

If the cross-sectional free area is large, the air flow caused by the first mode vibration spreads around the free end of the vibrating cantilever quite evenly (see Fig. 3). It is noted that the rigid plate is sufficiently far from the

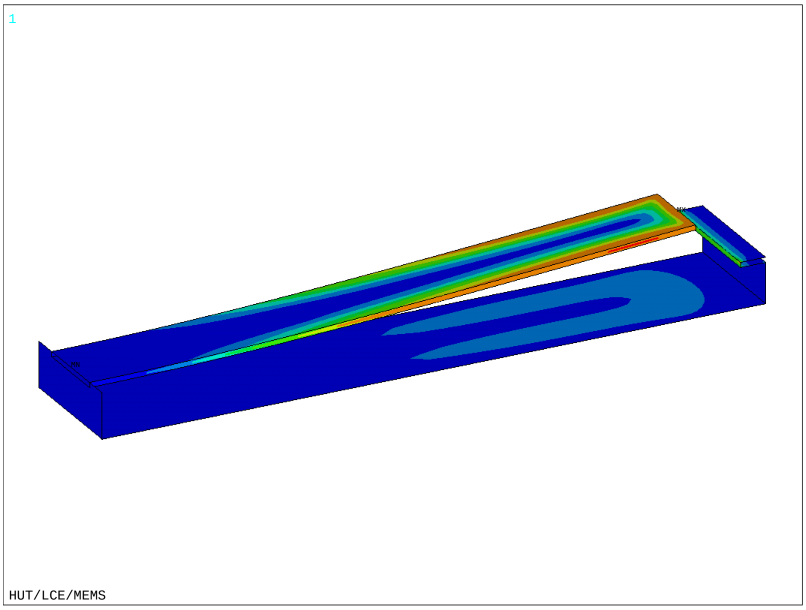

Figure 3: Nonuniform contours of the maximum of the air flow velocity in the cantilever system with air gap size of 256μm. The maximum velocity is 6.2m/s. Contours are scaled such that the smaller velocities are weighted more. The displacement is moved to its highest position in aim to show the contours better also in the air gap. The side walls are left out from the picture.

cantilever tip not to affect the air flow velocities substantially, and thus the main factor in the magnitude of this velocity is the displacement velocity of the cantilever. However, the side walls of the system are relatively close to the cantilever, and a minor flow blocking effect and thus an increase in the air flow velocity can be observed on the longitudinal sides of the cantilever tip. In general, the air flow velocity reaches its maximum at those points where the cross-sectional area of free flow is the smallest and the cantilever displacement is still considerable. For a small air gap system this smallest area is on the tip of the cantilever (Fig. 4). The maximum of the air flow velocity is reached when the cantilever displacement is at its minimum and the displacement velocity at its maximum. The direction of the air flow is opposite to the direction of the cantilever displacement.

A closer look to the laminar flow distribution in a small air gap structure shows up the typical behavior of the flow around the tip (Fig. 5). Most of the pressure difference induced air flow goes through the air gap and around the longitudinal sides of the cantilever to the back side of the progressive cantilever. Some of the air pushed away by the moving cantilever is cut by the rigid plate and forced to flow along the plate. Above the plate the reduced air flow is free, and below it the walls force the flow to turn and stay in the air film volume. Since the plate is wider than the vibrating cantilever, some of the cut portion of the flow dodges back around the transverse ends of the plate.

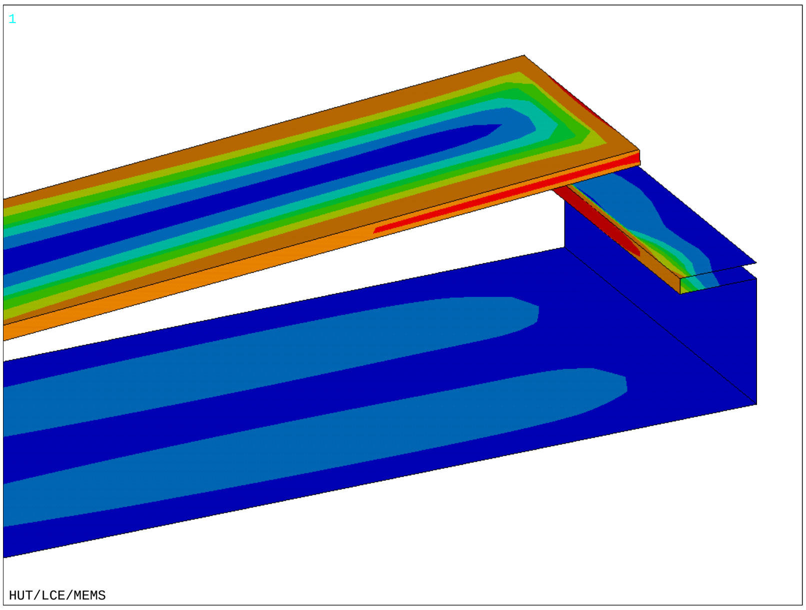

Figure 4: Non-uniform contours of the maximum of the flow velocity in the zoomed cantilever system with the air gap size of 4μm. The maximum velocity is 15.7m/s. The cantilever is displaced to its highest position to show the contours better also in the air gap. The side walls are not shown in this picture.

In the cantilever systems, we are discussing here, the air flow velocity is always larger than the maximum displacement velocity of the vibrating

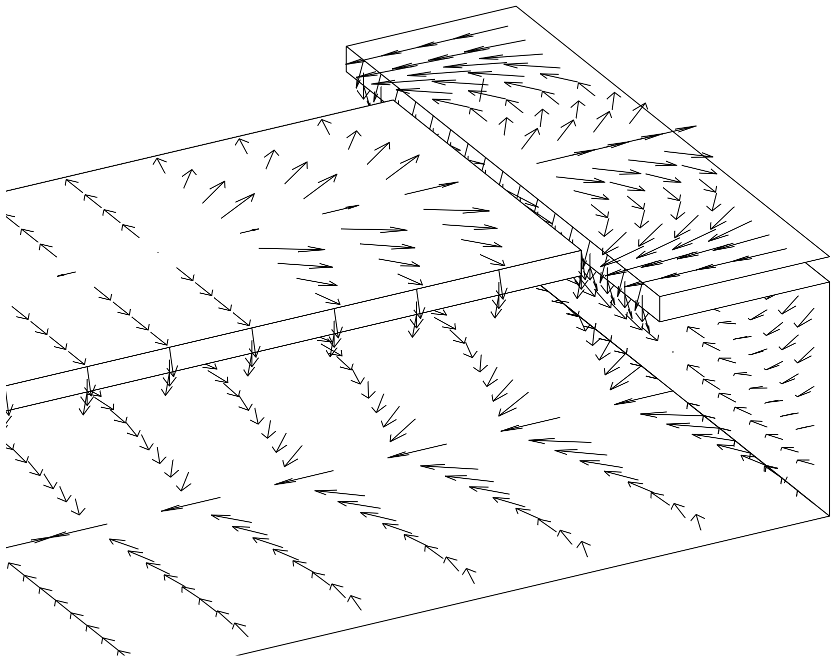

Figure 5: A schematic picture of the velocity vector field on top of the cantilever and rigid plate with air gap size of 4μm while the vibrating cantilever has just reached its maximum displacement velocity from the downwards position. The side walls are not shown in this picture.

cantilever (equation 3). This is due not only to the effect of the flow blocking of the rigid plate, walls and floor around the vibrating cantilever, but also due to the pressure difference between the front- and backside of the moving cantilever caused by the pushing of the air.

The air film volume underneath the cantilever, which is diminished or enlarged by the cantilever displacement, can only be vacated or filled, respectively, through the free air area beside the cantilever. Thus the pressure changes are larger in the closed air film than in the free volume above the cantilever. In addition, strong pressure differences cause large air flow velocities and large damping. The maximum pressure changes from the ambient pressure on the top of the vibrating cantilever reach only $44 - 55\%$ of the maxima under the cantilever. The narrower the free area around the vibrating cantilever, either between the cantilevers or on the longitudinal side of the vibrating cantilever, the higher the difference between the maximal pressure changes above and below the cantilevers is. In Figure 6 maximal air flow velocities and pressure ratio between the top and bottom parts of the cantilever are shown as a function of the air gap length. The length of the rigid plate does not affect the flow velocities substantially, only the size of the total free cross-sectional area and the displacement velocity of the cantilever are crucial for sufficient air volume transfer.

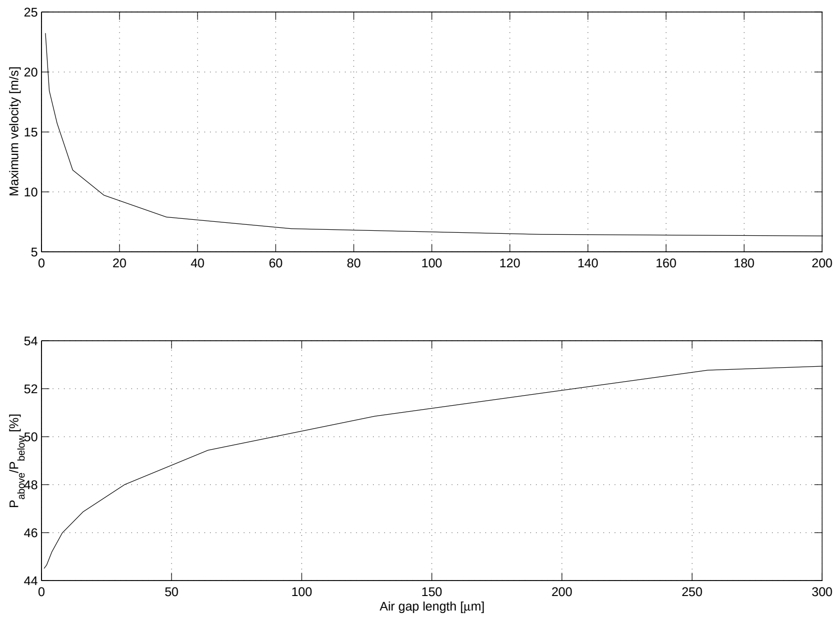

Figure 6: On the top graph the maximum air flow velocities on the cantilever system as function of the air gap size between the cantilever and rigid plate is shown. On the bottom graph the percentage ratio between the maximum pressure changes above the cantilever compared to the changes below it as function of the air gap is shown.

5 Conclusions

Linflow program has been designed to solve fluid-structure interaction in conventional cases, i.e., continuum size systems and for irrotational inviscid fluid. It uses boundary element method in the fluid flow evaluation, and thus the solution is generated only on the walls of the structure. The solution can, however, be expanded to plane cuts inside the flow area in need. The use of boundary element method expedites the solution procedure significally compared to conventional Navier-Stokes finite element method solution procedures in the whole volume.

The feasibility of Linflow in solution of micro-scale systems was studied. Due to the compatibility of Linflow with Ansys program, the model building even in three dimensions is easy. The structural problem can be solved using the Ansys solution methods, and the air effects are then solved by using Linflow, which has turned out to be relatively fast in deriving solutions.

The modelled small scale vibrating cantilever system was found to be sensitive to the changes in the cross-sectional free area between the lower and upper air volume part, especially in terms of pressure difference and flow velocity, which increase as the free area decreases. On the other hand, the resonance frequency of the damped vibration diminishes as a function of the decreasing free area. Linflow is able to predict damping rates in the eigenfrequencies relatively well, in fact within 0.6% compared to experimental results. The general flow and pressure field behaviour appeared rational.

However, the Linflow results did not predict nonlinear damping rate differences for different modes as the free area beside the vibrating cantilever decreased. Also the nonlinear behavior of very small scale systems, where the Knudsen number is large, did not occur. Thus, Linflow is an applicable tool for solving the fluid-structure interaction in micromechanical systems, whose dimensions are above the limit where squeezed film effects occur.

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References

[1] P. Ahrweiler and N. Gilbert. *Computer Simulations in Science and Technology Studies*, chapter 1, pages 1–13. Springer-Verlag, 1998.

[2] J. F. Wendt, editor. *Computational fluid dynamics, an introduction.* Springer-Verlag, 2^{nd} edition, 1996.

[3] J. van Kuijk. *Numerical Modelling of Flows in Micro Mechanical Devices.* PhD thesis, Twente University, Netherlands, June 1997.

[4] G. Beer and J. O. Watson. *Introduction to Finite and Boundary Element Methods for Engineers*, chapter 6–7, pages 141–282. Wiley & Sons, 1^{st} edition, 1992.

[5] L.E. Kinsler, A. R. Frey, A. B. Coppens, and J. V. Sanders. *Fundamentals of Acoustics*, chapter 3, pages 70–74. Wiley & Sons, New York, 3^{rd} edition, 1982.

[6] M. Gad-el-Hak. The fluid mechanics of microdevices-the freeman scholar lecture. *Journal of Fluids Engineering*, 121:5 – 33, March 1999.

[7] A.R. Paterson. *A first course in fluid dynamics*, chapter 16, pages 396–434. Cambridge University Press, 1^{st} edition, 1983.

[8] SGI Origin 2000. 128 r12k processors. 160 GB memory.

[9] J. Hietanen, J. Bomer, J. Jonsmann, W. Olthuis, P. Bergveld, and K. Kaski. Damping of a vibrating beam. Submitted to Sensors and Actuators, 1999.

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